[3] Kennedy, E. D., "Wake-like solutions of the laminar boundary-layer equations," AIAA J. 2, 225–231 (1964).
[4] Hayes, W. and Probstein, R., *Hypersonic Flow Theory* (Academic Press Inc., New York, 1959), p. 321.
[5] Lees, L. and Reeves, B., "Supersonic separated and reattaching laminar flows: 1. General theory and application to adiabatic boundary layer-shock wave interaction," Firestone Flight Science Lab., Graduate Aeronautical Lab., California Institute of Technology TR 3, p. 36–37 (October 1963).
[6] Hildebrand, F. B., *Introduction to Numerical Analysis* (McGraw Hill Book Co., Inc., New York 1956), p. 237.
[7] Clutter, D. W. and Smith, A. M. D., "Solution of the general boundary-layer equations for compressible laminar flow, including transverse curvature," Douglas Aircraft Div. Rept. LB31088, p. 34–37 (February 1963).

Parameters Affecting the Normal Shock Location in Underexpanded Gas Jets

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Introduction

Recently the authors have completed an experimental and theoretical determination of the axial position of the normal shock formed by an underexpanded supersonic nozzle for a variety of different exit conditions and working fluids. Lewis and Carlson[1] have provided a semiempirical correlation that appears to be adequate for the one particular shape of nozzle studied and for a static ambient. The correlation may not be appropriate for other nozzle shapes or for a supersonic ambient.

Effect of Exit-Flow Angle

Lewis and Carlson[1] studied two 15° half-angle conical nozzles. One had an area ratio (exit to throat) of 1.385 and the other had an area ratio of 3.868. They utilized N_2, CO_2, and He as the working fluids. They found an excellent correlation between their results and the expression

$$x/d_e = k_1 M_e \gamma_e^{1/2} (p_e/p_a)^{k_2} \qquad (1)$$

where x is the axial distance to the normal shock (measured from the nozzle exit plane), d_e is the nozzle exit diameter, M_e is the nozzle exit Mach number, γ_e is the ratio of specific heats at the nozzle exit, p_e is the static pressure at the nozzle exit, and p_a is the ambient static pressure. They found that $k_1 = 0.69$ and $k_2 = \frac{1}{2}$ provided a good fit to their data. Any effect of the exit flow angle is not provided explicitly in relation (1).

Recently[2–4] a number of additional theoretical and experimental determinations have been made for the axial distance to the normal shock utilizing both conically shaped nozzles and a nozzle with Foelsch coordinates.[5] The Foelsch shape is designed to provide axial flow (a zero flow angle) at the exit. The Foelsch nozzle was operated with two different gases ($\gamma_e = 1.225$ and $\gamma_e = 1.4$). The conical nozzles had both 6° and 15° half angles. The conical engines were operated with one gas, $\gamma_e = 1.225$. In addition, some shock positions were calculated (by the method of Bowyer, D'Attorre, and Yoshihara[6]) for conditions for which no experimental data were available.

It was discovered that, whereas the present data for a 15° half-angle conical nozzle can be correlated well with expression (1) with $k_1 = 0.69$ and $k_2 = \frac{1}{2}$, the Foelsch nozzle data falls outside the range of reasonable correlation with these values for the coefficients. The Foelsch data is correlated reasonably well by expression (1) if $k_1 = 0.813$ and $k_2 = \frac{5}{8}$. Only one data point was obtained for the 6° half-angle conical nozzle. It apparently falls outside the reasonable bounds of correlation for either the 15° conical or the Foelsch coordinate nozzles.

The result of this study suggests that the position of the normal shock is sensitive to the flow angle at the nozzle exit. This same conclusion was reached by Love.[7]

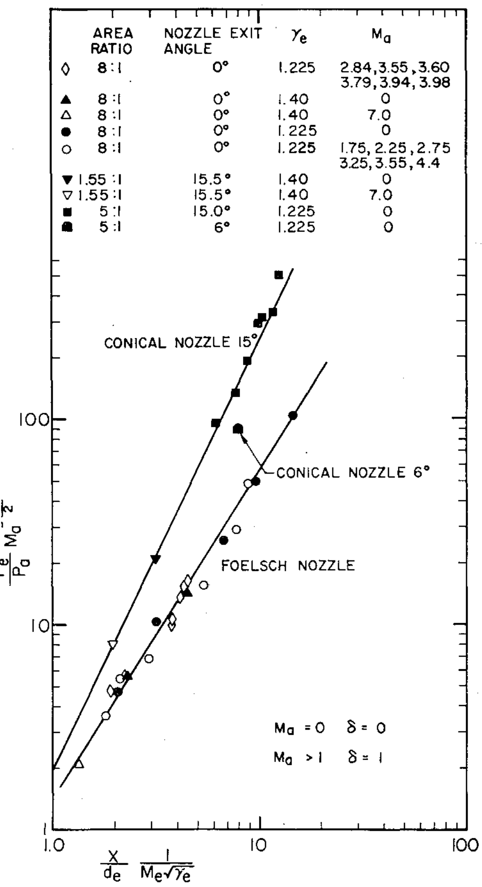

Fig. 1 Normalized experimental data for the axial position of the normal shock obtained from three different nozzle shapes.

Effect of a Supersonic Ambient

Experiments and calculations were conducted also with the 15° nozzle and the Foelsch nozzle with a supersonic ambient superimposed. As the ambient flow velocity was increased the distance to the normal shock decreased. A modification of relation (1) appears to be useful for correlating the data for conditions in which there is a supersonic ambient, i.e.,

$$x/d_e = k_1 M_e \gamma_e^{1/2} (p_e/p_a)^{k_2} M_a^{-1/2\delta k_2} \qquad (2)$$

where $\delta = 0$ when $M_a = 0$, and $\delta = 1$ when $M_a > 1$.

In Fig. 1 is given a correlation between the present experimental and theoretical results and relation (2) for the three different nozzle shapes and for a variety of subsonic and supersonic ambient conditions. A straight line has been drawn through the data obtained from the Foelsch nozzle and from the 15° half-angle conical nozzle representing the two sets of coefficients. There is some scatter in the data in Fig. 1. This may be due in part to uncertainties in the actual operating conditions and in part to the method of deducing the actual position of the normal shock from the original photographs. Despite the scatter, it appears that a reason-

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able correlation for supersonic conditions is obtained utilizing expression (2) for the two nozzle shapes studied.

Reference

[1] Lewis, C. H. and Carlson, D. J., "Normal shock location in underexpanded gas and gas-particle jets," AIAA J. **2**, 776–777 (1964).

[2] D'Attorre, L. and Harshbarger, F., "Experimental and theoretical studies of underexpanded jets near the Mach disc," General Dynamics/Astronautics Rept. GDA-DBE-64-008 (February 1964).

[3] D'Attorre, L. and Harshbarger, F., "Further experimental and theoretical studies of underexpanded jets near the Mach disc," General Dynamics/Astronautics Rept. GDA/DBE-64-041 (July 1964).

[4] D'Attorre, L. and Harshbarger, F., "The behavior of the Mach disc up to moderate pressure ratios," General Dynamics/Astronautics Rept. GD-DBE-64-042 (July 1964).

[5] Foelsch, K., "The analytical design of an axially symmetric laval nozzle for a parallel and uniform jet," J. Aeronaut. Sci. **16**, 161–166 (1949).

[6] Bowyer, J., D'Attorre, L., and Yoshihara, H., "Transonic aspects of hypervelocity rocket plumes," *Supersonic Flow, Chemical Processes and Radiative Transfer*, edited by D. B. Olfe and V. Zakkay (Pergamon Press, Ltd., London, 1964), pp. 203–209.

[7] Love, E. S., Grigsby, E. C., Lee, L. P., and Woodling, M. J., "Experimental and theoretical studies of axisymmetric free jets," NASA TR R-6 (1959).

Integrated Solar Pressure for a Transparent Balloon Satellite

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A BALLOON satellite is subject to an orbit perturbing force due to solar radiation pressure which is greater than the atmospheric drag force for medium- and high-altitude orbits. To minimize this perturbation, certain space experiments may require a transparent balloon that allows most of the solar radiation to pass through it. A method for calculating this force exerted on a transparent balloon satellite in terms of the index of refraction is given below. This method accounts for multiple reflections at the surface of the balloon satellite. Results of calculation are presented at the end of this paper.

Figure 1 shows the passage of a light ray through a balloon. Part of the ray is reflected at each interface and exerts a force that has a component parallel to the incident ray. This component, integrated over the surface, is the perturbing force. At each of the points A, B, C, etc. of Fig. 1, the ray breaks up into an infinite number of transmitted and reflected rays as shown by Fig. 2. The fraction reflected at the first surface is a function of the incidence angle i between the ray and the normal to the surface and is given by[1]

$$r = \frac{1}{2}\left[\frac{\sin^2(i - t)}{\sin^2(i + t)} + \frac{\tan^2(i - t)}{\tan^2(i + t)}\right] \qquad (1)$$

where r is the fraction reflected, i the incidence angle between ray and normal, and t is the transmittance angle between ray and normal.

The incidence angle i is related to the transmittance angle t by Snell's law:

$$\sin i = n \sin t$$

where n is the refractive index of the balloon material.

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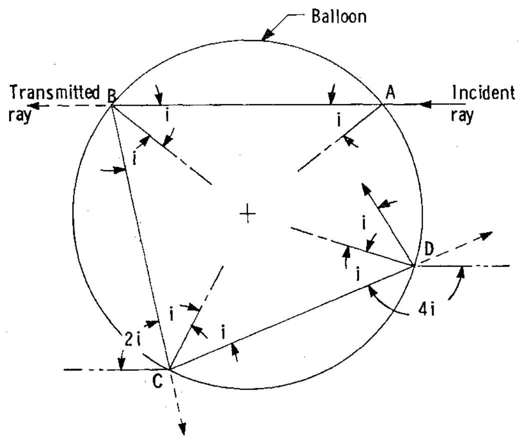

Fig. 1 Path of a light ray through a transparent spherical balloon satellite.

Since both the incident and reflected rays exert a component in the direction of the incident ray,[2] it is evident from the geometry of Fig. 2 that at each of the points a through d we have:

At point a:

Fraction reflected: r
Fraction transmitted: $1 - r$
Component parallel to incident ray: $r\,(1 + \cos 2i)$

At point b:

Fraction reflected: $r(1 - r)$
Fraction transmitted: $(1 - r)^2$
Component parallel to incident ray: $r(1 - r)[\cos(i - t) + \cos(i + t)]$

At point c:

Fraction reflected: $r^2(1 - r)$
Fraction transmitted: $r(1 - r)^2$
Component parallel to incident ray: $-r^2(1 - r)[\cos(i - t) + \cos(i + t)]$

At point d:

Fraction reflected: $r^3(1 - r)$
Fraction transmitted: $r^2(1 - r)^2$
Component parallel to incident ray: $r^3(1 - r)[\cos(i - t) + \cos(i + t)]$

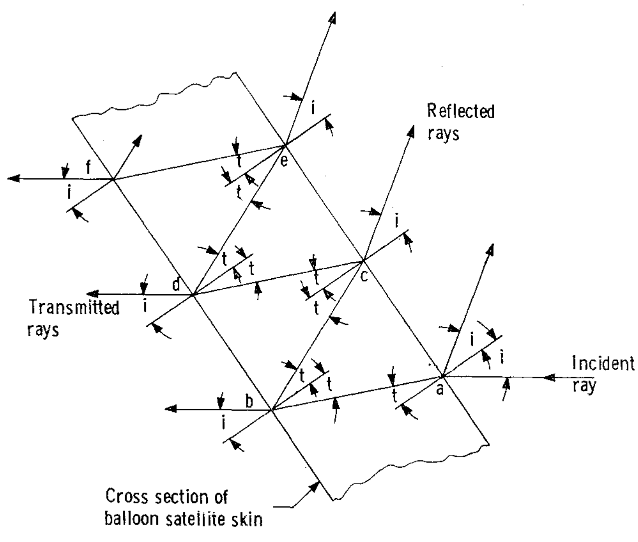

Fig. 2 Gemoetry of multiple reflection for a single incident ray.